An algorithm to make a resilient timetable

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Abstract
We propose a notion “resilience” of a timetable and introduce an algorithm to make a resilient timetable. We say a timetable is resilient if we can avoid giving significant inconvenience to passengers by doing rescheduling when a large delay occurs. Although resilience is a similar concept to robustness, only small delays are taken into account when we discuss robustness. Hence, rescheduling is not considered. In case of resilience, however, we deal with the cases when a large disruption occurs and rescheduling is done. We propose to measure resilience not only for a timetable but for a combination of a timetable, facilities such as tracks and rescheduling technique because they are very much relevant with the quality of rescheduling. When we define resilience, we assume the best effort is made for rescheduling, which means we can use an optimized rescheduling algorithm. We have developed an algorithm which produces a resilient timetable. The algorithm produces a timetable which minimized passengers’ disutility for a series of scenarios of accidents assuming the best rescheduling is done. We propose an MIP formulation of a timetabling algorithm in which optimized rescheduling algorithm is built in, which is one of our most significant contributions. We have implemented the algorithm on CPLEX and confirmed that our approach is very promising.

Keywords
Timetable, Resilience, Robustness, Rescheduling, MLIP

1 Introduction
Robustness of timetables has been paid a lot of attention in these days and a great number of papers on this subject have been published. Among them, some try to give a definition and/or a method to evaluate robustness(e.g. Takeuchi (2007), Carey (1999), Dewilde (2011)). For example, Takeuchi proposed an idea to define a robustness index of a train schedule as the expected value of the total increase in overall passenger disutility. There also exist papers which propose algorithms to get a robust timetable (e.g. Andersson (2013), Kroon (2008)). They assume small delays and want to find a more robust timetable by appropriately giving running time supplement and so on.

From the literature reviews, it seems that there is a general agreement that the term ”robustness” is used for small delays. When delays are small, we do not have to drastically change the original timetable. We do not have to cancel trains, we do not have to change train-set utilization plans and so on. In other words, we do not have to care about rescheduling process when we discuss robustness of a timetable.
On the other hand, serious accidents often happen. When a serious accident happens, train operation is suspended for some time. If a fatal accident occurs, train operation is suspended about an hour. In case of bad weather such as strong winds or heavy rain, the time of suspension might be more than a couple of hours. In such cases, we need to reschedule the train traffic so that passengers do not suffer from further inconvenience. For example, we partially cancel trains and turn back them so that we can continue transportation service in areas where we can still continue train operation. In addition, we need to change the tracks which trains use, departing orders, etc. so that we can get a feasible rescheduling plan.

It is strongly required by passengers to prepare a rescheduling plan which gives least further inconvenience to them. Qualities of rescheduled plan, however, highly depend on the original timetable. Some timetable is easy to reschedule whereas the others may be not. In addition, qualities of rescheduling depend on facilities and skills of dispatchers.

In this paper, we propose a notion "resilience" of a timetable and introduce an algorithm which makes a resilient timetable. What we have in our mind is that the timetable does not give significant inconvenience even when a large disruption occurs. We explicitly consider rescheduling. In addition, we measure resilience not only for a timetable but for a combination of a timetable, facilities such as tracks and rescheduling technique. When we measure resilience, we assume the best effort is made for rescheduling, which means we can use an optimized rescheduling algorithm for the measurement of resilience.

We have come up to this idea inspired by Liebchen (2009) which proposed an idea of "recoverable robustness." But we want to quantitatively evaluate resilience and establish an algorithm to get a resilient timetable. We have developed an algorithm which produces a resilient timetable based on the mixed integer programing formulation. The algorithm produces a timetable which minimizes passengers’ disutility for a series of scenarios of accidents assuming the best rescheduling is done. Although the purpose of our algorithm is to make a planned timetable, we built in an optimized rescheduling algorithm in it in order to evaluate rescheduling and calculate inconvenience of passengers, which is the most significant contribution of us. We have implemented the algorithm on CPLEX and confirmed by numerical experiments that our approach is very promising.

2 Resilience of timetable

2.1 Basic idea

We want to have a timetable which does not give significant inconvenience to passengers even when a big accident happens and rescheduling is done. We will explain our basic idea of resilience of a timetable (and other factors) using Figures 1 and 2.

In Figure 1, we assume that trains have to be partially cancelled and have to turn back from the station in the middle. Such rescheduling is very often done when we cannot continue train operation in a part of a line (in case of Figure 1, the lower part) due to a big accident, a natural disaster and so on. In this paper, we denote the planned timetable by dotted lines and the results of rescheduling by red, blue or black lines.

We think the timetable in Figure 1(a) is not so resilient. This is because when trains turn back at the station in the middle, definitely the turn back train is delayed. On the other hand, the turn back train can run on time if we adopt the timetable shown in Figure 1(b). Thus, the timetable of Figure 1(b) is more resilient than that of Figure 1(a).
Resilience is also influenced by facilities. We think the facility of Figure 2(b) gives a better influence concerning resilience than that of Figure 2(a). This is because there exist more switches in the facility of Figure 2(b), there is more freedom or flexibility of rescheduling (especially, turning back) in the facility.

2.2 Factors we have to consider

From the simple discussions in the previous section, we can now list up the factors we have to consider when we discuss resilience of timetables.

1. Timetable
   As we already observed in the previous section, some timetable is easy to reschedule but some are not. The timetable of Figure 1(a) is difficult to reschedule. It is quite difficult to operate the turn back trains on time, whereas in Figure 1(b), it is easy to operate the turn back trains on time. The difference exists in the timetables.

2. Facilities
   Among facilities, tracks and switches have a lot to do with rescheduling. There should exist proper number of tracks and switches. At the same time, they should be installed in proper locations.

3. Rescheduling
   There exist a variety of rescheduling technique, such as a (partial) cancellation of trains, a turn back of trains, a change of departing orders, a change of departing times, a change of tracks and so on.

   It is obvious that skills of rescheduling has a lot to do with resilience. If dispatchers have high skills of rescheduling, it is quite probable that a better rescheduling results which means a rescheduling by which passengers are not too much inconvenienced will be provided. On the other hand, if the dispatchers’ skill is low, the results must
be poor. The problem here is that when we discuss resilience, we do not know how high skill in rescheduling dispatchers possess.

4. Accidents

We have to be aware that the location where accidents happens has something to do with the rescheduling. The duration of the accidents is also relevant with the rescheduling.

2.3 Considering rescheduling - our approach

Takeuchi (2007) proposes to use simulation to estimate robustness of a timetable. They give a small disturbance and simulate the train traffic together with passengers’ behavior. In case of resilience, we have to consider that rescheduling is done. Thus, we have to estimate passengers’ inconvenience assuming some rescheduling is done. The problem is contents of rescheduling are different depending on the situation and the skills of dispatchers. In order to conquer this problem, we propose to assume that best efforts are done when we discuss resilience of timetables. Namely, we assume an optimal rescheduling is always done and this is obtained by using optimized rescheduling algorithm.

Another issue when we consider rescheduling is that we have to consider the location and the duration of accidents, which vary a lot from one case to another. Inspired by Liebchen (2009), we prepare scenarios, each of which corresponds to one case of an accident. Then we evaluate resilience for each scenario and calculate the sum of the evaluation values for the whole set of the prepared scenarios. We think we can prepare the scenarios based on the records of accidents which happened in the past.

Thus, resilience of a timetable (and other relevant factors) is defined by Formula (1).

\[
\sum_{sr \in SR} R_{sr}
\]  

3 An algorithm which makes a resilient timetable

3.1 Basic ideas

We first explain our approach to establish an algorithm to make a resilient timetable. What we would like to make is a planned timetable. If there is no disruption, trains run according to the planned timetable. Thus, it is desirable to minimize passengers’ traveling time when there is no disruption. But we also have to consider how much inconvenience passengers will suffer when an accident occurs and of course it is desirable to minimize the inconvenience. In this paper, we define a passenger’s disutility as the sum of the traveling time in normal situation and the increase of traveling time in case of a disruption. More concretely speaking, our objective function is expressed by Formula (2).

\[
\text{minimize } \alpha(\text{traveling time in plan}) + \beta(\text{inconvenience value})
\]
Here, “traveling time in plan” means the total sum of passengers’ traveling times when there is no disruption. “inconvenience value” means the total sum of increase of traveling times of all the passengers in case disruptions occur and rescheduling is done. \( \alpha \) and \( \beta \) are the weights.

We show the image of the overall construction of our algorithm in Figure 3. The algorithm is realized based on the mixed integer programming formulation. Although the purpose of our algorithm is to make a planned timetable, we built in an optimized rescheduling algorithm in it in order to evaluate rescheduling and calculate inconvenience. This idea is the most significant contribution of us.

The inputs to the algorithm are a set of scenarios and passengers’ data which means OD data of passengers.

Our algorithm could be regarded to consist of two parts; namely an algorithm for evaluation of inconvenience (Algorithm 1) and an algorithm to make a planned timetable (Algorithm 2). In addition, the base of these two algorithms are an optimized rescheduling algorithm (Algorithm 0) as shown in Figure 4. We can get Algorithm 1 by enhancing Algorithm 0 so that it produces an optimal solution for a set of scenarios. We can get Algorithm 2 by relaxing the constraints about the arrival / departure times and so on.

We will explain these algorithms one by one in the following sections. In Table. 1, we show the notations which we will use afterwards.

### 3.2 Algorithm 0: the rescheduling algorithm

When a big disruption occurs, we are compelled to suspend train operation in the damaged area. But we would like to continue train operation in other areas of the railway line by making trains turn back. We show an image of turning back of trains in Figure 5. This is considered to be a combination of partial cancelation of trains and changes of train-set utilization plans (sometime, changes of tracks are also necessary). This technique is very commonly done when it takes (say) more than an hour until we can resume normal train operation (see e.g. Amano (2015)).

Thus, in developing a rescheduling algorithm, turning back of trains has to be implemented together with other rescheduling technique such as a change of departing orders, a change of tracks, a change of train-set utilization plans etc.
In addition, we think it is very important to evaluate rescheduling from passengers’ viewpoint. Hence, as the objective function, we use inconvenience of passengers, which means the total sum of the increase of traveling times of all the passengers.

There are a lot of paper about train rescheduling algorithms (e.g. Schöbel (2001), Groth (2009), Pellegrini (2012), Törnquist (2012), Chigusa (2012), Tamura (2014), Sato (2013)). But none of them fit our purpose described above except that Sato (2013) is dealing with passengers’ inconvenience. Hence, we have developed our own rescheduling algorithm considering turning backs of trains and taking passengers’ inconvenience as the objective function.

Constraints about rescheduling

We show the constraints and the objective function of our rescheduling algorithm (Algorithm 0) from Formulae (3) to (39). This formulation can realize rescheduling in which not only turning backs of trains but almost all rescheduling technique such as a change of departing orders, a change of tracks, a change of train-set utilization plans, etc are considered.

\[
f^*_t + r^*_t \leq 1, \quad \forall t \in T, \forall s \in S \tag{3}
\]

\[
op^*_t \leq OP^*_t, \quad \forall t \in T, \forall s \in S_{op} \tag{4}
\]

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Formulae (3) - (5) are the constraints for the turning back trains. As we can see from Figure 5, in order to implement tuning back of trains, we should fix for each train, the segment of the railway line where the train runs. Formula (3) expresses the constraint that the same station cannot be selected as the departure station and as the terminal station at the same time. Formula (4) expresses that a train cannot run in the segment of the railway line which is not included in its original schedule. Formula (5) expresses a train starts its journey from a starting station and ends its journey at its terminal station. We can arbitrarily decide if a train should be canceled for each segment of the line by Formulae from (3) to (5). Formula (5) are the constraints for outbound trains and we also prepare constraints for inbound trains (likewise hereafter).

Formulae (6) and (7) are the constraints for a change of a departing order. Formula (6) is about the order of a train itself and means there does not exist an order with itself. Formula (7) decides the departing order, which means either train $t$ and train $^t$ departs first. We exclude this constraint for the same train but in order to consider the conflict of tracks at the station where trains turn back, we also apply this constraint for the trains whose directions are opposite. By Formulae (6) and (7), we can arbitrarily decide the departing orders of trains.

Formulae (8) - (11) are the constraints for train-set utilization plans, which play a very important role in rescheduling. Formula (8) expresses a constraint that a train-set is not assigned to a train whose direction is the same. Formula (9) means that only one train-set is assigned to a train or when a train-set begins the duty, there is no predecessor (a preceding train). We give $dp_t^s$ as one of the inputs to the algorithm which means where the train-sets exist. Formula (10) means that a train-set has a successor (a succeeding train) or the train-set ends the duty. Formula (11) defines the number of duties. By Formulae (8) - (11), we can arbitrarily make the duties of train-sets.

Formulae (12) and (13) are the constraints for the change of times of trains. Formula (12) expresses that a train has to depart after the planned departure time and Formula (13) expresses that a train arrives after the planned arrival time. In our algorithm, we do not allow trains depart or arrive before the planed times. By Formulae (12) and (13), we can arbitrarily fix the times of trains for stations.
Constraints imposed by physical conditions, which must be satisfied.

- \( a_t^{s+1} - d_t^s \geq MR_{down}^s - M(1 - op_t^s), \forall t \in T_{down}, s \in S_{op} \) (15)
- \( d_t^s - a_t^s \geq MST_{down}^s - M(2 - op_t^{s-1} - op_t^s), \forall t \in T_{down}, s \in S_{way} \) (16)
- \( d_t^s - a_t^s \geq MH^s - M(1 - oo_t^s), \forall t, i \in T, s \in S \) (17)
- \( a_t^{s+1} - a_t^s \geq HW^s - M(3 - od_{t,i}^s - op_t^s - op_t^s), \forall t, i \in T, s \in S \) (19)
- \( a_t^s - d_t^s \geq HW^s - M(5 - od_{t,i}^s - op_t^{s-1} - op_t^s - op_t^{s-1} - op_t^s), \forall t, i \in T, s \in S \) (20)
- \( a_{t_i} - d_{t_i} \geq LI - M(3 - od_{t,t'} - oo_{t,t'}^s), \forall t, i, t', i \in T, s \in S_{way} \) (21)
- \( a_{t_i} - d_{t_i} \geq LI - M(5 - od_{t,i} - op_t^{s-1} - op_t^s - op_t^{s-1} - op_t^s), \forall t, i \in T, s \in S_{way} \) (22)
- \( a_{t_i} - d_{t_i} \geq LI - M(5 - od_{t,t'} - oo_{t,t'}^s - oo_{t,t'} - tr_{t,s}^n - tr_{t',s}^n), \forall t, i, t', i \in T, s \in S_{edge}, \forall t \in TN \) (23)

Formulae (15) - (17) are the constraints about running times and dwell times. Formula (15) expresses the running time of a train must be larger than the technically minimum running time. Likewise, Formula (16) is a constraint about dwell times and Formula (17) is a constraint about turn back times.

Formulae (18) - (20) are the constraints about headways. Formula (18) is for the arrival - arrival headway and Formula (19) and (20) are the constraints for departure - departure and arrival - departure headway, respectively.

Formulae (21) - (23) are the constraints about occupation of tracks in a station. Formula (21) expresses a constraint that the track used for turning back is occupied from the arrival of the train to its departure. Likewise, Formulae (22) and (23) are the constraints for an intermediate station and a terminal station respectively. Again, Formulae (21) - (23) are for Figure 6 and we can deal with other types of track layouts by slightly changing these constraints.
Constraints about passengers’ behavior

In order to estimate inconvenience of passengers, we need to grasp the route (meaning selections of trains by a passenger) of each passenger. The trains which passengers take in normal condition and their traveling times are constants but in the following formulation we treat them as variables because we want to enhance the algorithm in the following section.

\[ AV^t_p \leq OP^t_p + M(1 - PS_{p}^t), \quad \forall p \in P, \forall t \in T, \forall s \in S_{op} \]  
\[ AV^t_p \geq (D^t_p - PT_p)/M - M(1 - PD^t_p), \quad \forall p \in P_{down}, \forall t \in T_{down}, \forall s \in S_{op} \]  
\[ AV^t_p \leq (D^t_p - PT_p)/M + 1 + M(1 - PD^t_p), \quad \forall p \in P_{down}, \forall t \in T_{down}, \forall s \in S_{op} \]  
\[ \sum_{t \in T_{down}} GT^t_p = 1, \quad \forall p \in P_{down} \]  
\[ GT^t_p \leq (1 - \sum_{t \in T_{down}, t \leq t} AV^t_p)/M + 1, \quad \forall p \in P_{down}, \forall t \in T_{down} \]  
\[ TT^t_p \geq \Lambda^{t+1} - PT_p - M(2 - GT^t_p - PA^{t+1}), \quad \forall p \in P_{down}, \forall t \in T_{down}, \forall s \in S_{op} \]  
\[ ap^t_p \geq PS_{p}^t - M(1 - gt^t_p), \quad \forall p \in P, \forall t \in T, \forall s \in S_{op} \]  
\[ d^t_p \geq PT_p - M(2 - gt^t_p - PS_{p}^t), \quad \forall p \in P_{down}, \forall t \in T_{down}, \forall s \in S_{op} \]  
\[ \sum_{t \in T_{down}} gt^t_p = 1, \quad \forall p \in P_{down} \]  
\[ tt_p \geq a^{t+1} - PT_p - M(2 - gt^t_p - PS_{p}^t), \quad \forall p \in P_{down}, \forall t \in T_{down}, \forall s \in S_{op} \]  
\[ ic_p \geq tt_p - TT_p, \quad \forall p \in P \]  
\[ ic_p \geq 0, \quad \forall p \in P \]

Formulae (24) - (30) are the constraints for passengers’ behavior in the normal situation. Formulae (24) - (26) specifies that a passenger can only take a train which runs (at least) a part of the passenger’s route and the departure time is larger than the time of the passenger’s appearance. Formula (27) expresses a constraint that a passenger can choose only one train when he/she select a train to take and by Formulae (28) and (29), a train which departs earliest among the trains which could be taken by the passenger is chosen. Formula (30) calculates the traveling time of a passenger by subtracting the time of appearance from the arrival time at the destination station.

Likewise, Formulae (31) - (34) are the constraints concerning passengers’ behavior in case rescheduling is done. Finally, Formula (35) calculates the difference of traveling times between the normal situation and the situation when rescheduling is done. Formula (36) is used to ignore the case when traveling time in case of rescheduling is smaller than that of normal situation.

How to give information about disruption

Because we assume a disruption lasts for a certain time, we need to devise how to give this information as a constraint. In other words, it is not suffice to give such information as a delay of one train.
Formulae (37) and (38) are the constraints for the case in which train operation is not possible between two stations for a certain time of period and trains could start from the stations of the both ends only after the certain time has passed.

**Objective function**

The objective function which expresses the increase of traveling times is depicted by Formula (39).

\[
\text{minimize } \sum_{p \in P} ic_p
\]  

(39)

### 3.3 Algorithm 1: Evaluating inconvenience for a set of scenarios

We enhance Algorithm 0 so that we can deal with a set of scenarios. The differences are as follows:

- For a scenario \(sr \in SR\), we enhance all the variables in Table 1 except \(AV_p^t\) and \(GT_p^t\) which are the variables about passengers’ behavior in normal situation to each scenario. We also enhance the constraints of Formulae (3) - (23), (31) - (36) which contain those variables for each scenario. For example, Formula (3) is rewritten as Formula (40).

\[
f_{t,s}^{sr} + r_{t,s}^{sr} \leq 1, \quad \forall t \in T, \forall s \in S, \forall sr \in SR
\]  

(40)

- We define how to give information of disruption for each scenario as described in Formulae (37), (38).

- We change the objective function to Formula (41), which means the minimization of increase of traveling times in the results of rescheduling for all the scenarios. Because each scenario is mutually independent, this is the same as the minimization for each scenario.

\[
\sum_{p \in P, sr \in SR} ic_p^{sr}
\]  

(41)

### 3.4 Algorithm 2: Algorithm to make a timetable which minimizes the total sum of traveling times

When we make a (planned) timetable, it is indispensable to take passengers’ convenience into account. Among all, minimization of traveling times is considered to be the most important. We construct an algorithm to make a planned timetable which minimizes the total sum of the traveling times of passengers by slightly modifying Algorithm 0.

The differences from Algorithm 0 are as follows:
We change the constants concerning the planned timetable into variables and eliminate variables concerning rescheduling. In more detail, we change the constants $OP_t^s$, $D_t^s$ and $A_t^s$ in Table 1 to variables and eliminate variables $op_t^s$, $d_t^s$ and $a_t^s$. In accordance with this modification, we change $op_t^s$, $d_t^s$ and $a_t^s$ in Formulae (5), (15)-(23) to $OP_t^s$, $D_t^s$, $A_t^s$.

For the departing order of trains, we change Formula (7) to Formulae (42) - (44). We assume that trains of the same direction run following their train number.

$$od_{t,i} + od_{i,t} = 1, \quad \forall t \in T_{down}, \forall i \in T_{up}$$

$$od_{t,i} = 1, \quad \forall t, i \in T_{down}, t < i$$

$$od_{t,i} = 1, \quad \forall t, i \in T_{down}, t > i$$

We eliminate the constraints of rescheduling, ie. Formulae (4), (12) and (13), which are not necessary any more.

We eliminate the constraints of passengers’ behavior in case of rescheduling. That is, we eliminate variables $gt_i^t$, $tt_i^t$, $ic_p$ in Table 1 and related constraints; Formulae (31) - (36).

We eliminate the constraints for the information of disruption; Formulae (37) and (38).

We change the objective function as Formula (45), which means minimization of traveling times in case of a normal situation.

$$\sum_{p \in P} TT_p$$

3.5 An algorithm which makes the most resilient timetable

By combining Algorithm 1 and Algorithm 2 and giving some modifications, we can now construct an algorithm which makes the most resilient timetable for a given set of scenarios. The modifications are as follows:

- We enhance all the variables in Table 1 except $AV_t^p$ and $GT_t^p$ to each scenario. In accordance with this modification, we enhance the constraints Formulae (3) - (23), (31) - (36) which contain these variables for each scenario.

- For each scenario, we define how to give disruption as Formulae (37) and (38).

- We change the constants concerning the planned timetable $OP_t^s$, $D_t^s$ and $A_t^s$ to variables. In accordance with this, we set up constraints by changing $op_t^s$, $d_t^s$ and $a_t^s$ to $OP_t^s$, $D_t^s$, $A_t^s$ in Formulae (5), (15) - (23).

- For the variables $od_{i,t}^s$, $f_t^s$, $r_t^s$, $oo_{i,t}^s$, $dp_t^s$, $fw_t^s$ and $tr_{t,s}^n$, we prepare variables $OD_{i,t}^s$, $F_t^s$, $R_t^s$, $OO_{i,t}^s$, $DP_t^s$, $FW_t^s$, $TR_{t,s}^n$ for the planned timetable. We prepare another constraints for the planned timetable similar to Formulae (3), (5)-(11), (14). But for the departing orders, we use similar constraints as Formulae (42)-(44).
We change the objective function to Formula (46) following Formula (2).

\[ \alpha \sum_{p \in P} TT_p + \beta \sum_{p \in P, sr \in SR} i_{c_p}^{sr} \]  

(46)

4 Numerical Experiments and Discussions

4.1 Numerical Experiments

We have conducted numerical experiments to confirm if our algorithm works properly. The track layout we assumed is the same as that of Figure 6. We show the passengers’ OD data we used in Table 3. There are 20 groups of passengers. We assumed that we are requested to set up six trains at the maximum. We assumed that three train-sets are available; two at Station A and one at Station B, respectively. We set \( \alpha \) and \( \beta \) in Formula (46) as 1.0 and 0.2 respectively.

We show the result of our resilient timetabling algorithm in Figure 7. In making this timetable, we prepared three scenarios. We show the scenarios together with the optimized rescheduling for each scenario in Figure 8 - Figure 10. We show in Table 2 the value of the objective function of the resilience together with those of the algorithm for the planned timetable and the algorithm for rescheduling for each scenario.

The processing time needed was 2 minutes 49 seconds.

![Figure 7: Output - resilient timetable.](image)

4.2 Discussions

We have succeeded to obtain the most resilient timetable for a given assumption of the number of trains, the number of available train-set, the passenger OD data and the scenarios of accidents. We can understand the timetable shown in Figure 7 is very resilient from the rescheduling results of Figure 8 - Figure 10.

We have to confirm if our algorithm works on a bigger problem because Figure 7 is just a toy problem. We need to increase the numbers of trains, stations, passengers and scenarios in order to solve more practical problems and confirm if our algorithm works for larger problems within a reasonable processing time.
5 Conclusion

We have proposed a notion of resilience of a timetable. What we have in our mind is we would like to make a timetable which does not give significant inconvenience even when a large disruption occurs. We define resilience of a timetable for a combination of a timetable, facilities and rescheduling. We proposed an idea to define resilience by the disutility of passengers when a large disruption occurs and rescheduling is done. In estimation of resilience, we showed an idea to assume the best rescheduling is done for the prepared set of scenarios which contain possible cases of accidents.

Based on this idea, we have developed an algorithm which makes the most resilient timetable using the mixed integer programming formulation. The algorithm produces a
timetable which minimized passengers’ disutility for a series of scenarios of accidents assuming the best rescheduling is done. One of our contributions is that we devised a formulation of a timetabling algorithm in which an optimized rescheduling algorithm is built in.

We have implemented the algorithm on CPLEX and through numerical experiments, we have confirmed that our approach is very promising.

For our future works, we intend to confirm if our algorithm works well for practical problems whose sizes are larger.

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Table 1: Notations.

<table>
<thead>
<tr>
<th>Sets</th>
<th>Description</th>
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<td>$S$</td>
<td>Set of stations</td>
</tr>
<tr>
<td>$S_{\text{edge}}$</td>
<td>Set of terminal stations</td>
</tr>
<tr>
<td>$S_{\text{way}}$</td>
<td>Set of intermediate stations</td>
</tr>
<tr>
<td>$S_{\text{op}}$</td>
<td>Set of operation sections</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of trains</td>
</tr>
<tr>
<td>$T_{\text{down}}$</td>
<td>Set of outbound trains</td>
</tr>
<tr>
<td>$T_{\text{up}}$</td>
<td>Set of inbound trains</td>
</tr>
<tr>
<td>$T_{\text{N}}$</td>
<td>Set of track numbers</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of passengers</td>
</tr>
<tr>
<td>$P_{\text{down}}$</td>
<td>Set of outbound passengers</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constants</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP$_{st}$</td>
<td>1 if a train $t$ runs between station $s$ and station $(s + 1)$ in plan. 0 otherwise</td>
</tr>
<tr>
<td>$D_{st}^t$</td>
<td>Planned departure time of train $t$ at station $s$</td>
</tr>
<tr>
<td>$A_{st}^t$</td>
<td>Planned arrival time of train $t$ at station $s$</td>
</tr>
<tr>
<td>MR$^t_{dr}$</td>
<td>Minimum running time between station $s$ and station $(s + 1)$ in direction $dr$</td>
</tr>
<tr>
<td>MST$_{dr}^{st}$</td>
<td>Minimum dwell time at station $s$ in direction $dr$</td>
</tr>
<tr>
<td>MSH$_{st}^d$</td>
<td>Minimum turning back time at station $s$</td>
</tr>
<tr>
<td>HW$_s$</td>
<td>Minimum headway of station $s$</td>
</tr>
<tr>
<td>LI</td>
<td>Minimum interval of tracks</td>
</tr>
<tr>
<td>PD$_{sp}$</td>
<td>1 if passenger $p$ departs from station $s$. 0 otherwise</td>
</tr>
<tr>
<td>PA$_{sp}$</td>
<td>1 if passenger $p$ finally arrives at station $s$. 0 otherwise</td>
</tr>
<tr>
<td>PS$_{sp}$</td>
<td>1 if passenger $p$’s route contains from station $s$ to station $(s + 1)$. 0 otherwise</td>
</tr>
<tr>
<td>PT$_p$</td>
<td>Appearance time of passenger $p$</td>
</tr>
<tr>
<td>RE</td>
<td>Time when operation is resumed</td>
</tr>
<tr>
<td>M</td>
<td>Huge number</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>op$^t_{st}$</td>
<td>1 if a train $t$ runs between station $s$ and station $(s + 1)$ in rescheduling. 0 otherwise</td>
</tr>
<tr>
<td>$d^s_t$</td>
<td>Departure time of train $t$ from station $s$ (in rescheduling)</td>
</tr>
<tr>
<td>$a^s_t$</td>
<td>Arrival time of train $t$ at station $s$ (in rescheduling)</td>
</tr>
<tr>
<td>$od_{st}$</td>
<td>1 if a train $t$ departs earlier than a train $\ell$ of the same direction. 0 otherwise</td>
</tr>
<tr>
<td>$f^s_t$</td>
<td>1 if a train $t$ begins its journey from station $s$. 0 otherwise</td>
</tr>
<tr>
<td>$r^s_t$</td>
<td>1 if a train $t$ ends its journey at station $s$. 0 otherwise</td>
</tr>
<tr>
<td>$oc_{st}$</td>
<td>1 if a train $t$ turns back as train $\ell$ at station $s$. 0 otherwise</td>
</tr>
<tr>
<td>$dp^s_{st}$</td>
<td>1 if a train-set assigned to train $t$ begins its journey from station $s$. 0 otherwise</td>
</tr>
<tr>
<td>$fu^s_{st}$</td>
<td>1 if a train-set assigned to train $t$ terminates its duty at station $s$. 0 otherwise</td>
</tr>
<tr>
<td>$tr_{tn}$</td>
<td>1 if a train $t$ uses track number $tn$ at station $s$. 0 otherwise</td>
</tr>
<tr>
<td>$At^t_p$</td>
<td>1 if passenger $p$ can get on train $t$. 0 otherwise</td>
</tr>
<tr>
<td>$GT^t_p$</td>
<td>1 if passenger $p$ gets on train $t$ in the plan. 0 otherwise</td>
</tr>
<tr>
<td>$gt^t_p$</td>
<td>1 if passenger $p$ gets on train $t$ in the rescheduling. 0 otherwise</td>
</tr>
<tr>
<td>$TT^t_p$</td>
<td>Passenger $p$’s traveling time in the plan</td>
</tr>
<tr>
<td>$tt^t_p$</td>
<td>Passenger $p$’s traveling time in the rescheduling</td>
</tr>
<tr>
<td>$ic^t_p$</td>
<td>Passenger $p$’s increase of traveling time</td>
</tr>
</tbody>
</table>
Table 2: Scenarios and results.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value of objective function (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travelling time</td>
<td>467</td>
</tr>
<tr>
<td>Scenario 1: Station B - Station C 60 minutes suspension</td>
<td>207</td>
</tr>
<tr>
<td>Scenario 2: Station B - Station C 10 minutes suspension</td>
<td>0</td>
</tr>
<tr>
<td>Scenario 3: Station A - Station B 30 minutes suspension</td>
<td>127</td>
</tr>
<tr>
<td><strong>Value of Resilience</strong></td>
<td>533.8</td>
</tr>
</tbody>
</table>

Table 3: Passengers’ OD data.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Depart time</th>
<th>Depart sta.</th>
<th>Arrival sta.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>0:02</td>
<td>St. A</td>
<td>St. C</td>
</tr>
<tr>
<td>Down</td>
<td>0:07</td>
<td>St. A</td>
<td>St. B</td>
</tr>
<tr>
<td>Down</td>
<td>0:12</td>
<td>St. A</td>
<td>St. C</td>
</tr>
<tr>
<td>Down</td>
<td>0:17</td>
<td>St. A</td>
<td>St. B</td>
</tr>
<tr>
<td>Down</td>
<td>0:22</td>
<td>St. A</td>
<td>St. C</td>
</tr>
<tr>
<td>Down</td>
<td>0:27</td>
<td>St. A</td>
<td>St. B</td>
</tr>
<tr>
<td>Down</td>
<td>0:32</td>
<td>St. A</td>
<td>St. C</td>
</tr>
<tr>
<td>Down</td>
<td>0:37</td>
<td>St. A</td>
<td>St. B</td>
</tr>
<tr>
<td>Down</td>
<td>0:42</td>
<td>St. A</td>
<td>St. C</td>
</tr>
<tr>
<td>Down</td>
<td>0:47</td>
<td>St. A</td>
<td>St. B</td>
</tr>
<tr>
<td>Up</td>
<td>0:02</td>
<td>St. B</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:07</td>
<td>St. C</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:12</td>
<td>St. B</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:17</td>
<td>St. C</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:22</td>
<td>St. B</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:27</td>
<td>St. C</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:32</td>
<td>St. B</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:37</td>
<td>St. C</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:42</td>
<td>St. B</td>
<td>St. A</td>
</tr>
<tr>
<td>Up</td>
<td>0:47</td>
<td>St. C</td>
<td>St. A</td>
</tr>
</tbody>
</table>